# GENERAL SECTION SECONDARY 2 PURE MATHIMATICS



# Geel 2000 Language Schools Math Department First Term

2024 / 2025

Name
Class

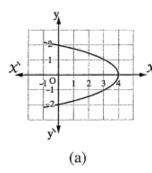
# (Unit 1)

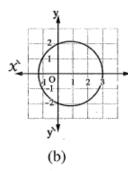
## **Real functions**

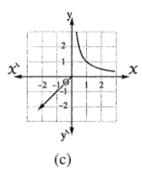
# (Domain, Range and monotony)

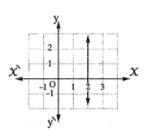
### 1-

(1) Which of the following figures represents a function of X?









- (2) The opposite figure represents a function of X whose domain
  - (a) R

- (b)  $\mathbb{R} ]-1$ ,2[
- (c)  $\mathbb{R} [-1, 2]$

is .....

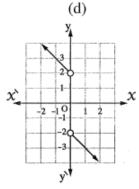
- (d)  $\mathbb{R} \{0\}$
- (3) The opposite figure represents a function of X whose range is ......
  - (a)  $\mathbb{R} [0, 2]$
- (b)  $\mathbb{R} \{0\}$
- (c)  $\mathbb{R} [0, 2[$
- (d)  $\mathbb{R} [0, 2]$
- (4)  $f(x) = \sqrt{4-x^2}$ , then the domain of the function  $f = \cdots$

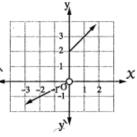


(b) 
$$]-2,2[$$

(c) 
$$[-2,2[$$

(d) 
$$]-2,2]$$





# Operations on functions (composition functions)

2-

Choose the correct answer from those given:

- (1)  $f(x) = \frac{1}{x}$ ,  $g(x) = \sqrt{x}$ , then the domain of  $(f, g) = \dots$ 
  - (a)  $\mathbb{R} \{0\}$
- (b) R
- (c)  $\mathbb{R}^+$  (d)  $[0, \infty[$
- (2) f(X) = X + 1,  $g(X) = X^2$ , then  $(f \circ g)(2) = \dots$ 
  - (a) 3
- (b) 4
- (c) 5
- (d) 9
- (3) The domain of the function  $f: f(x) = \sqrt{5-x}$  equals .....
  - (a)  $\mathbb{R} \{5\}$  (b)  $\mathbb{R}^+$
- (c)  $]-\infty$ , 5] (d)  $[5,\infty[$
- (4)  $f(X) = \sqrt{X}$ ,  $g(X) = X^2$ , then the domain of  $(f \circ g) = \cdots$ 
  - (a)  $[0, \infty[$  (b)  $\mathbb{R}$
- (c) R+
- (d) R

3-

If  $f(X) = \frac{1}{x}$ , g(X) = X + 3, find:

- $(1)(f \circ g)(X)$
- $(2)(g \circ f)(X)$

and state the domain in each case.

4-

If  $f(X) = \frac{1}{X}$ , g(X) = X + 3, find:

- $(1)(f \circ g)(X)$
- $(2)(g \circ f)(X)$

and state the domain in each case.

# Properties of functions

5-

Find the type of each function whether it is even , odd or otherwise :

(1) 
$$f(x) = \frac{x^3}{|x|+2}$$
 (2)  $f(x) = \sin x^2 - \sin^2 x$ 

6-

[b] If f(x) = x - 1,  $g(x) = \sqrt{x}$ , then find  $(g \circ f)(x)$  and determine its domain, then find  $(g \circ f)(5)$ 

7-

Draw the graph of the function 
$$f: f(x) = \begin{cases} x |x| & \text{when } x < 0 \\ \frac{x^4}{|x|} & \text{when } x > 0 \end{cases}$$

, then deduce its domain and discuss its type whether it is even , odd or otherwise.

## Graphical (basic and piecewise)

8-

Graph the function  $f: f(X) = 4 - (X - 2)^2$ , then deduce its range, its monotony and whether the function is odd, even or otherwise.

9-

Graph the function 
$$f: [-2, 6] \longrightarrow \mathbb{R}$$
 where  $f(x) = \begin{cases} 4-x & , & -2 \le x < 1 \\ x & , & 1 \le x \le 6 \end{cases}$  and from the graph deduce its range and discuss its monotonicity.

10-

Graph the function 
$$f: f(X) = \begin{cases} X-1, & 2 < X \le 4 \\ -1, & -2 \le X \le 2 \end{cases}$$
 from the graph determine its range.

11-

**[b]** If 
$$f: [-4,3] \longrightarrow \mathbb{R}$$
,  $f(X) = \begin{cases} 4 & \text{when } X < 0 \\ (X-1)^2 + 1 & \text{when } 0 \le X \le 3 \end{cases}$ 

, graph the function f , then from the graph ,  $\mathbf{deduce}$  :

- (1) The range.
- (2) The monotonicity.
- (3) The type (even, odd, otherwise).

### Geometric transformations

12-

Use the graph of the function f where  $f(X) = X^2$  to represent the function g where  $g(X) = (X - 1)^2 + 2$  and from the graph determine the range of the function g and discuss its monotonicity and tell whether it is even, odd or otherwise.

13-

Use the curve of the function f where  $f(X) = X^3$  to represent each of the following functions:

(1) 
$$f_1(x) = (x+1)^3$$

$$(2) f_2(X) = X^3 + 1$$

## Solving absolute value equations and inqualities

14-

Find in  $\mathbb R$  the solution set for each of the following :

$$(1)|2x-5| \le 3$$

15-

Find in  $\mathbb{R}$  the S.S. of the following equation algebraically:  $\sqrt{x^2-4x+4}=4$ 

16-

Find in  $\mathbb R$  the S.S. of the following inequality algebraically :  $|x-3| \ge 5$ 

17-

Find the solution set in  $\mathbb{R}$  of each of the following :

$$(1)|2X-3|+|6-4X| \le 0$$

$$(2)|2x-4|=x+3$$

18-

Find graphically in  $\mathbb{R}$  the solution set of the inequality: |5-x|>3

19-

Find in  $\mathbb{R}$  the solution set of the inequality :  $\sqrt{4 x^2 - 12 x + 9} > 5$ 

20-

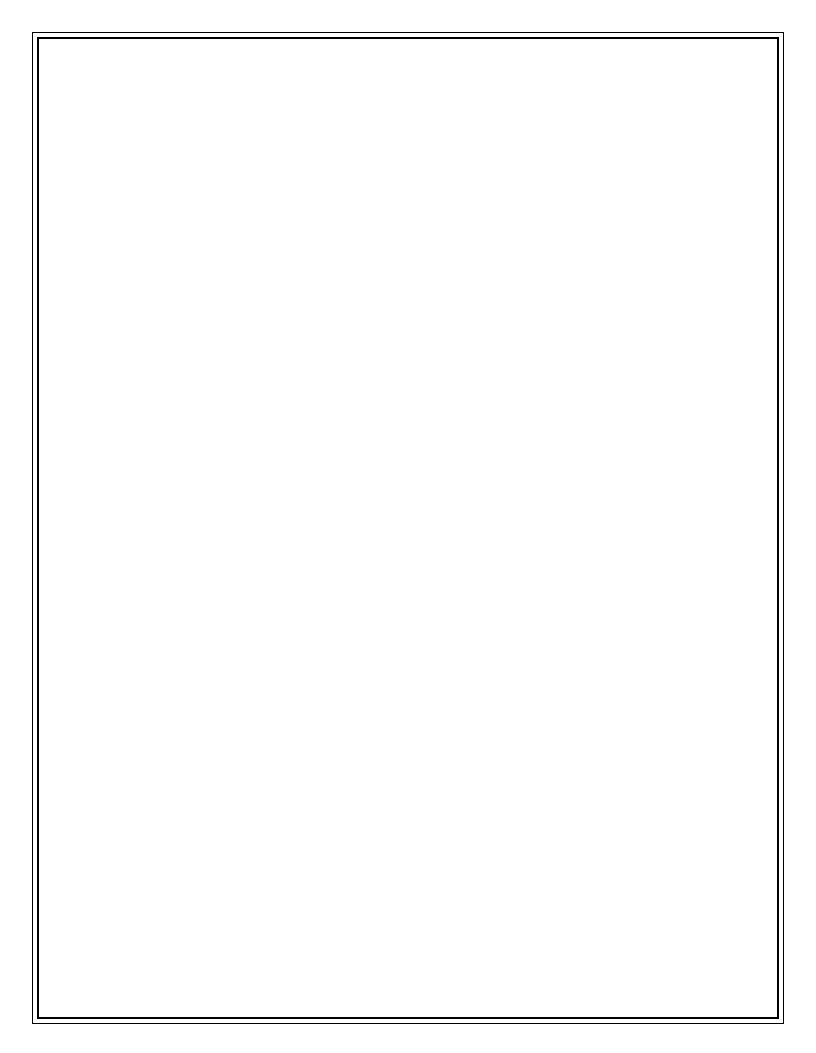
Find in  $\mathbb{R}$  the solution set of each of the following :

$$(1)|x+2|+5=9$$

$$(2)|2X-3| \le 5$$

21-

Solve in  $\mathbb{R}$  the equation : 2|x-2|-|2-x|=3



# (Unit 2)

# Rational exponents and exponintial equations

1-

Find the value of X which satisfies:  $2^{x+1} = 25$  by using calculator.

2-

Find in  $\mathbb{R}$  the solution set of each of the following:

$$\left(\frac{1}{2}\right)^{X+1} + \left(\frac{1}{2}\right)^{X+3} + \left(\frac{1}{2}\right)^{X+5} = 84$$

3-

Simplify:

$$\frac{16^{x+\frac{1}{4}} \times 9^{x+2}}{8^{x-1} \times 18^{x+2}}$$

4-

Find the S.S. of each of the following in  $\mathbb R$ :

$$4^{x} + 2^{x+1} = 8$$

5-

Put in the simplest form :  $\frac{9^{4 \text{ n}+1} \times 4^{2-2 \text{ n}}}{3^{9 \text{ n}+1} \times 48^{1-n}}$ 

6-

If  $f(2 X) = 3^X$ , solve in  $\mathbb R$  the equation : f(2 X + 4) + f(2 X) = 90

# **Applications**

7-

A patient gets 40 mg. of medicine , his body gets rid of 10 % of this medicine every hour.

- (1) Write the exponential function which represents the quantity of medicine left after t hours.
- (2) Estimate this quantity of medicine left in the body after 4 hours.

8-

The number of cows in a cattle farm is 80 cows and the reproduction rate of these cows is 18 % annually. Find the number of cows after 4 years, given by  $C = a (1 + r)^t$  where t is the number of years, a is the starting amount and r is the annual percentage of reproduction.

### The invers function

9-

The opposite figure represents a function  $f: X \longrightarrow Y$ 

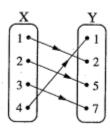
• then 
$$f^{-1}(2) = \cdots$$

(a) 5

(b) 1

(c) 3

(d) 4



10-

If  $f(x) = 3 + \sqrt{2x-1}$ , find:  $f^{-1}(x)$  and find the domain of  $f^{-1}$  and its range.

11-

If f(x) = x - 3, then find the inverse function of f

12-

Find the inverse function of f where  $f(x) = x^3 + 1$ 

13-

If f(X) = 5 X, find  $f^{-1}(X)$  and represent it graphically.

## Logarithmic function and its properties

14-

Without using calculator , find the value of :  $\log 25 + \frac{\log 8 \times \log 16}{\log 64}$ 

15-

#### Find in $\mathbb{R}$ the S.S of the equation :

$$\log_7 X + \log_7 (X + 6) = 1$$

16-

If 
$$X y = 16$$
, prove that:  $3 \log_2 X + 4 \log_2 y - \log_2 X y^2 = 8$ 

17-

Find in  $\mathbb R$  the solution set of each of the following :

$$\log_5(X^2 - 25) - \log_5(X - 5) = 2$$

18-

Find the S.S. in  $\mathbb R$  of the equation :

$$(\log_3 x)^2 - 2\log_3 x - 3 = 0$$

19-

Simplify: 
$$(1)\frac{1}{\log_a ab} + \frac{1}{\log_b ab}$$

20-

Find the S.S. of the following equation in  $\mathbb{R}$ :  $\log_2 x + \log_2 (x+1) = 1$ 

Without using calculator, find the value of the following:

$$2 \log 25 + \log \left(\frac{1}{3} + \frac{1}{5}\right) + 2 \log 3 - \log 30$$

22-

Solve the equation in  $\mathbb{R}$ :  $\log_2 x + \log_2 (x+1) = 1$ 

23-

Solve in  $\mathbb{R}$  the following equation :  $\log_4 x = 1 - \log_4 (x - 3)$ 

24-

Find in  $\mathbb R$  the solution set for each of the following :

$$\log_3 x + \log_x 3 = 2$$

25-

( 1 ) Find in  $\mathbb R$  the solution set of the equation :  $\log_3 x + \log_\chi 3 = 2$ 

(2) Prove that: 
$$\frac{\log 729 - \log 64}{\log 9 - \log 4} = 3$$

26-

| **Prove that**: 
$$\log_2 \frac{4}{11} - \log_2 \frac{7}{130} + \log_2 \frac{77}{65} = \log_5 125$$

27-

Find in  $\mathbb{R}$  the S.S. of the equation :

$$\log_4 X = 1 - \log_4 (X - 3)$$

28-	
<b>Prove that:</b> $\log_b a \times \log_c b \times \log_d a$	$c \times \log_a d = 1$

# (Unit 3) Introduction of limits of functions

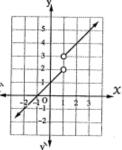
1-

#### Choose the correct answer from those given:

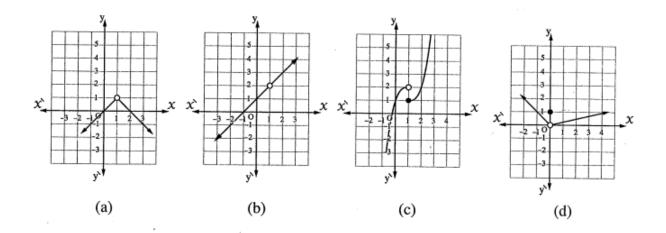
(  ${\bf 1}$  ) The opposite figure represents the graph of the function f , then

$$\lim_{x \longrightarrow 1} f(x) = \dots$$

- (a) 2
- (b) 3
- (c) 1
- (d) not exist



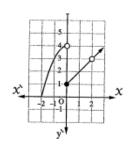
(2) Which of the following functions has no limit at x = 1?



2-

## In the opposite figure , find :

- $(1) f (zero^+)$
- (2) f (zero<sup>-</sup>)
- **(3)** f (2)
- (4)  $\lim_{x \to 2} f(x)$



# Finding tke limit of the function algebrically

3-

Choose the correct answer from those given:

(1) 
$$\lim_{x \to 0} \frac{1+x}{4x-1} = \dots$$
  
(a) -1 (b)  $\frac{1}{4}$  (c)  $-\frac{1}{4}$   
(2)  $\lim_{x \to 3} \frac{x^2-9}{x-3} = \dots$ 

(b) 
$$\frac{1}{4}$$

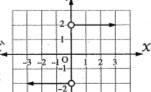
(c) 
$$-\frac{1}{4}$$

$$(a) - 6$$

(3) The opposite figure represents f(x)

Then: 
$$\lim_{x \to 2} f(x) = \dots$$
 (a) 0 (b) -2 (c) 2 (d) not exist

(a) 0 
$$x \rightarrow 2$$



4-

Find:

$$\lim_{x \to 3} \frac{x^2 - x - 6}{x - 3}$$

5-

Find:

$$\lim_{x \to -1} \left( \frac{5 x^2 + 5 x}{3 x^2 - 3} \right)$$

6-

$$\lim_{x \to 0} \frac{x^2 + x}{x} = \dots$$

Find: 
$$\lim_{x \to 1} \frac{x^3 - 2x^2 + 1}{x^2 + x - 2}$$

(2) 
$$\lim_{x \to 2} \frac{x^2 - 5x + 6}{x^3 - 8}$$

9-

$$\lim_{x \to 5} \frac{x-5}{2x-3}$$

10-

Find: 
$$\lim_{x \to 2} \frac{x^3 + 8}{x + 2}$$

11-

$$\lim_{x \longrightarrow -3} \frac{x^2 + 4x + 3}{x^2 - 9}$$

Find: (1) 
$$\lim_{x \to -2} \frac{x^3 + 8}{x + 2}$$

The law

13-

$$\lim_{x \to 4} \frac{\sqrt{x-3}-1}{x-4}$$

14-

Find: (1) 
$$\lim_{x \to -2} \frac{(x+3)^5 - 1}{x+2}$$

15-

Find: (1) 
$$\lim_{x \to 2} \frac{x^3 + 8}{x + 2}$$

16-

| Find: (1) 
$$\lim_{x \to 3} \frac{x^5 - 243}{x^2 - 9}$$

17-

Find: 
$$\lim_{x \to 5} \frac{(x-3)^5 - 32}{x-5}$$

Find: (1) 
$$\lim_{x \to -3} \frac{x^4 - 81}{x^5 + 243}$$

Find: (1) 
$$\lim_{x \to -2} \frac{\sqrt[3]{x} + \sqrt[3]{2}}{x+2}$$

20-

Find: (1) 
$$\lim_{x \to \sqrt{5}} \frac{x^7 - 125\sqrt{5}}{x^4 - 25}$$

21-

$$\lim_{x \longrightarrow 5} \frac{(x-3)^5 - 32}{x^2 - 5x}$$

22-

$$\lim_{x \to -3} \frac{x^4 - 81}{x^5 + 243}$$

23-

$$\lim_{x \to -3} \frac{x^4 - 81}{x^5 + 243}$$

$$\lim_{x \longrightarrow 2} \frac{x^7 - 128}{x^5 - 32}$$

Find: (1) 
$$\lim_{x \to 2} \frac{2x^4 - 32}{x^2 - 4}$$

Find: (1) 
$$\lim_{x \to \frac{1}{3}} \frac{27 x^4 - \frac{1}{3}}{3 x - 1}$$

# Limit of the function at infinity

27-

Find: (1) 
$$\lim_{x \to \infty} \frac{2x^3 - 5x}{x^4 + 3}$$

28-

$$\lim_{x \to \infty} \frac{2 x^3 - 9}{|3 x|^3 + 7}$$

29-

$$\lim_{x \to \infty} \frac{5 x^{-3} + 4 x^{-2} - 3}{7 x^{-3} - 2 x^{-2} + 8}$$

30-

Find: (1) 
$$\lim_{x \to \infty} \left( \sqrt{4x^2 - 2x + 1} - 2x \right)$$

31-

$$\lim_{x \to \infty} \frac{2x^2 - x + 1}{x^3 - x^2 + 1}$$

Find: (1) 
$$\lim_{x \to \infty} \frac{2x^3 - 3}{3x^2 + 1}$$

$$\lim_{x \to \infty} \frac{x^{-2} + 3}{x^{-3} + 6}$$

34-

Find: (1) 
$$\lim_{x \to \infty} \frac{\sqrt{9 x^2 + 3}}{6 x - 1}$$

35-

$$\lim_{x \to \infty} \frac{5 - 6x - 3x^2}{2x^2 + x + 4}$$

36-

$$\lim_{x \to \infty} \frac{-x}{\sqrt{4+x^2}}$$

37-

Find: (1) 
$$\lim_{x \to \infty} \frac{x^3 - 4x + 5}{(2x - 1)^3}$$

Find: (1) 
$$\lim_{x \to \infty} \frac{5 - 6x - 3x^2}{2x^2 + x + 4}$$

Find: (1) 
$$\lim_{x \to \infty} \frac{3 x^2 + x - 1}{8 x^2 - 3}$$

40-

$$\lim_{x \to \infty} \frac{2 x^{-1} - 3 x^{-2}}{4 + x^{-1}}$$

41-

$$\lim_{x \to \infty} \frac{3 x + \sqrt{4 x^2 + 5}}{5 x - 3}$$

42-

$$\lim_{x \to \infty} \left( \sqrt{x^2 + 5x} - x \right)$$

$$\lim_{x \to \infty} \frac{x^3 - 2}{|x|^3 + 1}$$

Find: (1) 
$$\lim_{x \to \infty} \frac{x^3 - 4x + 5}{(2x - 1)^3}$$

45-

Find: (1) 
$$\lim_{x \to \infty} (\sqrt{x^2 + 5x} - x)$$

$$\lim_{x \to \infty} \frac{1}{x} \sqrt{3 + 4x^2}$$

# Limits of tragnometric functions

47-

$$\lim_{x \longrightarrow 0} \frac{1 - \cos^2 2 x}{3 x^2}$$

48-

$$\lim_{x \to 0} \frac{x - x \cos x}{\sin^2 3 x}$$

49-

$$\lim_{x \to 0} \frac{\sin 3 x - \sin 2 x}{5 x}$$

50-

Find: (1) 
$$\lim_{x \to 0} \frac{2x + \sin 3x}{\tan 5x}$$

51-

Find: 
$$\lim_{x \to 0} \frac{x^3 + \sin 3x}{5 x \cos 2x}$$

Find: (1) 
$$\lim_{x \to 0} \frac{x - x \cos x}{\sin^2 3 x}$$

$$\lim_{x \to 0} \frac{x^2 + \sin 3 x}{5 x \cos 2 x}$$

54-

$$\lim_{x \to 0} \frac{\sin^2 3 x - \tan 2 x^2}{x^2 \cos 4 x}$$

55-

$$\lim_{x \to 0} \frac{x \tan 2 x \cos 3 x}{x^2 + \sin^2 3 x}$$

56-

Find: (1) 
$$\lim_{x \to 1} \frac{\sin(x-1)}{x^2 + x - 2}$$

Find: (1) 
$$\lim_{x \to 0} \frac{x - x \cos x}{\sin^2 3 x}$$

Find: (1) 
$$\lim_{x \to 0} \frac{1 - \cos x + \sin x}{1 - \cos x - \sin x}$$

59-

$$\lim_{x \to 0} \frac{\sin 3 x - \tan 2 x}{5 x}$$

60-

$$\lim_{x \to 0} \frac{\sin 2 x + 5 \sin 3 x}{x}$$

$$\lim_{x \to 0} \frac{x \tan x + \sin^2 3 x}{2 x^2 + \sin 3 x^2}$$

## Existence of the limit of a piecewise function

62-

Find: (1) 
$$\lim_{x \to \pi} f(x)$$
 where  $f(x) =\begin{cases} \frac{2 \sin x}{\pi - x} &, & x < \pi \\ 1 - \cos x &, & x > \pi \end{cases}$ 
(2)  $\lim_{x \to \infty} x \tan x^{-1}$ 

63-

$$f(X) = \begin{cases} \frac{X \tan X + \sin^2 3 X}{5 X^2} \\ 2 \cos 2 X \end{cases}$$

64-

Discuss the existence of the limit of the function f where

$$f(x) = \begin{cases} \frac{1 - \cos x}{x} &, & x > 0 \\ 2 \sin x &, & x \le 0 \end{cases} \text{ at } x = 0$$

Find: 
$$\lim_{x \to 3} f(x)$$
 if  $f(x) = \begin{cases} x^2 + 1 &, & x < 3 \\ 3x + 1 &, & x > 3 \end{cases}$ 

66-

$$| \text{If } f(X) = \begin{cases} x^2 - 2X & , & -2 < X < 1 \\ 3X - 4 & , & 1 \le X < 4 \end{cases}$$

, discuss the existence of each of the following:

$$(1)$$
  $\underset{x \longrightarrow -2}{\text{Lim}} f(x)$ 

$$(2) \lim_{x \to 1} f(x)$$

67-

If 
$$\lim_{x \to 2} f(x) = 7$$
 where  $f(x) = \begin{cases} x^2 + 3 \text{ m}, & x < 2 \\ 5x + k, & x > 2 \end{cases}$ , find m and k

68-

Discuss the existence of Lim f(X) which  $X \longrightarrow 0$  where

$$f(x) = \begin{cases} \frac{5 x^2 + \tan 2 x^2}{\sin^2 x}, & x > 0\\ 7 \cos 3 x, & x < 0 \end{cases}$$

Find: (1) 
$$\lim_{x \to \pi} f(x)$$
 where  $f(x) =\begin{cases} \frac{2 \sin x}{\pi - x} &, & x < \pi \\ 1 - \cos x &, & x > \pi \end{cases}$ 
(2)  $\lim_{x \to \infty} x \tan x^{-1}$ 

70-

If 
$$f(x) =\begin{cases} \frac{x^2 - 7x + 12}{x - 3}, & x > 3\\ ax - 7, & x < 3 \end{cases}$$

,  $\lim_{x \to 3} f(x) = -1$ , then find the value of a

## Continuity

70-

Discuss the continuity of the function f

where 
$$f(x) = \begin{cases} 8 - x & , & x \ge 3 \\ x + 2 & , & x > 3 \end{cases}$$
 at  $x = 3$ 

71-

Discuss the continuity of the function f where:

$$f(x) = \begin{cases} \frac{x^2 + x - 2}{x + 2} &, & x > -2 \\ 3x + 5 &, & x \le -2 \end{cases}$$
 at  $x = -2$ 

72-

Discuss the continuity of the function f on its domain where:

$$f(x) = \begin{cases} 1 + \sin x & \text{when } 0 \le x \le \frac{\pi}{2} \\ 1 - \cos 2x & \text{when } x > \frac{\pi}{2} \end{cases}$$

73-

Discuss the continuity of the function f:

$$f(x) = \begin{cases} \frac{\sin(x-2)}{x^2 - 4} &, & x < 2 \\ 1 - \frac{3}{x^2} &, & x \ge 2 \end{cases}$$

Discuss the continuity of 
$$f: f(x) = \begin{cases} \frac{x^2 - x - 6}{x - 3}, & x < 3 \\ 2x - 1, & x \ge 3 \end{cases}$$

75-

## Discuss the continuity of the function f where

$$f(X) = \begin{cases} \frac{\sin X}{2 x - 2 \pi} &, & x < \pi \\ \frac{1}{2} \cos x &, & x \ge \pi \end{cases} \text{ at } x = \pi$$

76-

# Discuss the continuity of f at x = 1

where 
$$f(x) =\begin{cases} x^2 + 3 & , & x \ge 1 \\ \frac{x^2 + 2x - 3}{x - 1} & , & x < 1 \end{cases}$$

77-

# Find the value of a which makes the function f

where 
$$f(x) = \begin{cases} \frac{x^2 - 5x + 6}{x^3 - 8} & \text{, } x \neq 2 \\ \frac{-2}{|a|} & \text{, } x = 2 \end{cases}$$
 continuous at  $x = 2$ 

Find the value of k which makes the function f continuous at x = 2 where

$$f(x) = \begin{cases} \frac{x^2 + x - 6}{x^3 - 8} & , & x \neq 2 \\ \frac{2}{|\mathbf{k}|} & , & x = 2 \end{cases}$$

79-

Discuss the continuity of 
$$f: f(x) = \begin{cases} x^2 + 3 & \text{,} & x \ge 1 \\ \frac{x^2 + 2x - 3}{x - 1} & \text{,} & x < 1 \end{cases}$$

80-

Discuss the continuity of the function f

where 
$$f(X) = \begin{cases} X^2 + 3 & \text{,} & X \ge 1 \\ \frac{X^2 + 2X - 3}{X - 1} & \text{,} & X < 1 \end{cases}$$

81-

Find the value of the constant a if the function f:

where 
$$f(x) = \begin{cases} \frac{(x+3)^4 - 81}{x}, & x \neq 3 \\ a, & x = 3 \end{cases}$$
 is continuous at  $x = 3$ 

Discuss the continuity of the function  $\boldsymbol{f}$  where :

$$f(x) = \begin{cases} \frac{\sin x}{\pi - x} &, & x \neq \pi \\ 1 &, & x = \pi \end{cases}$$
 at  $x = \pi$ 

# (Unit 4) The sine rule

1-

Solve  $\triangle$  ABC in which m ( $\angle$  B) = 35°, m ( $\angle$  C) = 70°, and the diameter length of its circumcircle = 32 cm.

2-

In  $\triangle$  ABC, if m ( $\angle$  A) = 35°, a = 17 cm. and b = 20 cm.

**Prove that:**  $\triangle$  ABC has two solutions, then find them.

3-

Find the perimeter of  $\triangle$  ABC in which m ( $\angle$  A) = 57° 13°, c = 8.7 cm. and m ( $\angle$  B) = 64° 18°

4-

ABC is a triangle in which:  $m (\angle A) = 35^{\circ}$ , a = 8 cm. and b = 6 cm. Find:  $m (\angle B)$ 

5-

ABC is a triangle in which: b = 12 cm.,  $m (\angle B) = 75^{\circ}$  and  $m (\angle C) = 45^{\circ}$  Find:

(1) a

(2) The area of  $\triangle$  ABC

(3) The radius length of the circumcircle of the triangle ABC

ABC is a triangle in which m ( $\angle$  A): m ( $\angle$  B): m ( $\angle$  C) = 3:4:3 If a = 5 cm., find the perimeter of  $\triangle$  ABC

7-

Solve the triangle ABC in which a = 8 cm.,  $m (\angle A) = 60^{\circ}$  and  $m (\angle B) = 40^{\circ}$ 

8-

Solve the acute-angled triangle ABC in which a = 21 cm., b = 25 cm. and the diameter length of its circumcircle = 28 cm.

9-

Find the shortest side length in  $\triangle$  ABC , in which : m ( $\angle$  A) = 43° , m ( $\angle$  B) = 70° and c = 9 cm. Find the area of the triangle ABC.

10-

ABC is a triangle in which: AC = 4.7 cm.,  $m (\angle B) = 34^{\circ}$  and  $m (\angle C) = 66^{\circ}$ Find the length of  $\overline{BC}$ , then find the area of its circumcircle.

11-

ABC is a triangle in which m ( $\angle A$ ) = 40°, a = 5 cm. and b = 7 cm. Find m ( $\angle B$ ) approximating to the nearest minute.

12-

Solve the triangle ABC in which a = 5 cm. b = 7 cm. and  $m (\angle C) = 65^{\circ}$ 

ABC is a triangle in which m ( $\angle$  A) = 85°, m ( $\angle$  B) = 55° and c = 5 cm. Find the area of the circumcircle of  $\triangle$  ABC

#### The cosine rule

14-

The perimeter of the triangle ABC is 52 cm., a = 13 cm. and b = 17 cm. Find the measure of the greatest angle.

15-

Solve the triangle ABC in which a = 5 cm. b = 7 cm. and  $m (\angle C) = 65^{\circ}$ 

16-

Solve the triangle ABC in which a = 9 cm., b = 7 cm. and c = 5 cm., then find its area.

17-

In  $\triangle$  ABC, if a=4 cm., b=5 cm. and c=6 cm., prove that:  $\cos C=\cos 2$  A, then find the circumference of the circumcircle of  $\triangle$  ABC

18-

Solve the triangle ABC in which a = 9 cm., b = 15 cm. and  $m (\angle C) = 106^{\circ}$ 

19-

Solve the triangle ABC in which a = 15 cm., b = 13 cm. and c = 14 cm.

ABC is a triangle in which a = 27 cm., b = 35 cm. and c = 18 cm. Find the measure of the greatest angle.

## Solution of the triangle

21-

In  $\triangle$  ABC,  $\cos A = \frac{2}{5}$ , b = 2.5 cm. and c = 2 cm.

**Prove that:**  $\triangle$  ABC is an isosceles triangle and find its area.

22-

In  $\triangle$  ABC, if m ( $\angle$  A) = 35°, a = 17 cm. and b = 20 cm.

**Prove that:**  $\triangle$  ABC has two solutions, then find them.

23-

### In the opposite figure:

ABCD is a quadrilateral in which

$$AB = 8 \text{ cm.}, BC = 6 \text{ cm.}, m (\angle B) = 90^{\circ}$$

, DC = 5 cm. and m (
$$\angle$$
 ACD) =  $60^{\circ}$ 

Find the area of the circumcircle of  $\triangle$  ADC

C 60 60 B 8 cm. A

In  $\triangle$  ABC , show whether it has only 1 solution , 2 solutions or non , given your answer to the nearest decimal :

(1) m (
$$\angle$$
 B) = 110°, b = 8 cm. and c = 5 cm.

(2) m (
$$\angle$$
 A) = 60°, a = 7 cm. and b = 9 cm.

25-

Show whether the triangle ABC has one, two or no solution, given that:  $m (\angle A) = 100^{\circ}$ , a = 12 cm. and b = 15 cm.

26-

ABC is a triangle in which m ( $\angle A$ ) = 52°, a = 21 cm. and b = 26 cm.

**Prove that:**  $\triangle$  ABC has two solutions, then find them.

27-

ABC is a triangle in which:  $\frac{\sin A}{3} = \frac{\sin B}{4} = \frac{\sin C}{5}$  and its perimeter = 24 cm. Find its area.